Statistics
Fall 2022
Lecture 7

Combination $\quad {}_nC_r = \dfrac{n!}{r!\cdot(n-r)!}$

${}_7C_3 = \dfrac{7!}{3!\cdot(7-3)!} = \dfrac{7!}{3!\cdot 4!} = \dfrac{7\cdot\cancel{6}\cdot 5\cdot\cancel{4!}}{\cancel{3\cdot 2}\cdot 1\cdot\cancel{4!}} = \dfrac{35}{1} = \boxed{35}$

7 $\boxed{\text{MATH}}$ $\boxed{\rightarrow}$ PRB $\boxed{\downarrow}$ $\boxed{{}_nC_r}$ 3 $\boxed{\text{Enter}}$

Find ${}_{250}C_8 \approx \boxed{3.4\times10^{14}}$

Suppose we have 8 Dimes & $\boxed{\text{12 Nickels}}$

$\boxed{\text{How many ways}}$ can we select 3 dimes and $\boxed{\text{5 Nickels}}$. No replacement, order does not matter.

Dimes $\cdot$ Nickels

${}_8C_3 \cdot {}_{12}C_5 = \boxed{44352}$

Suppose we have a deck of cards, 25 cards, 7 face cards, and 3 Aces.

Draw 3 cards, No replacement, order does not matter.

1) Total # of Selections: $_{25}C_3 = \boxed{2300}$

2) # of Selections for 2 face cards & 1 Ace.

$_7C_2 \cdot {}_3C_1 = \boxed{63}$

3) $P(\text{2 Face Cards \& 1 Ace}) = \dfrac{_7C_2 \cdot {}_3C_1}{_{25}C_3} = \boxed{\dfrac{63}{2300}}$

4) $P(\text{1 Face and 2 Aces}) = \dfrac{_7C_1 \cdot {}_3C_2}{_{25}C_3} = \boxed{\dfrac{21}{2300}}$

5 Females & 13 Males.

We need 4 people, No replacement, order does not matter

$P(\text{4 Females}) = \dfrac{_5C_4 \cdot {}_{13}C_0}{_{18}C_4} = \dfrac{5}{3060} = \boxed{\dfrac{1}{612}}$

$P(\text{4 Males}) = \dfrac{_5C_0 \cdot {}_{13}C_4}{_{18}C_4} = \dfrac{715}{3060} = \boxed{\dfrac{143}{612}}$

$P(\text{2F \& 2M}) = \dfrac{_5C_2 \cdot {}_{13}C_2}{_{18}C_4}$

$= \dfrac{780}{3060} = \boxed{\dfrac{13}{51}}$

$\boxed{P(\text{at least 1}) = 1 - P(\text{None})}$

$P(\text{at least 1 Female}) = 1 - P(\text{No Female})$

$= 1 - P(\text{All Males}) = 1 - \dfrac{143}{612} = \boxed{\dfrac{469}{612}}$

$P(\text{at least 1 Male}) = 1 - P(\text{No Males})$

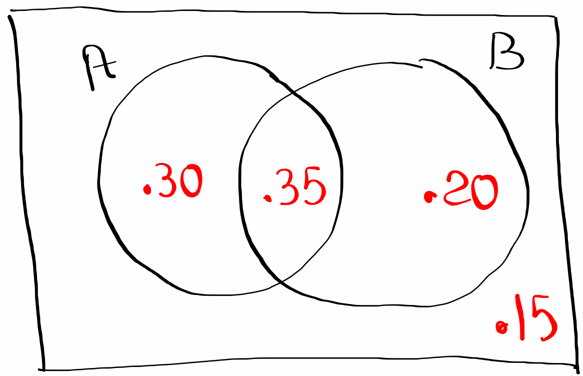
$= 1 - P(\text{All Females}) = 1 - \dfrac{1}{612} = \boxed{\dfrac{611}{612}}$

Suppose

$P(A) = .65$ ✓

$P(B) = .55$ ✓

$P(A \text{ and } B) = .35$



Conditional Prob.:

Total = 1

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{.35}{.65} = \boxed{\frac{7}{13}} = \boxed{.538}$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.35}{.55} = \boxed{\frac{7}{11}}$$

$P(\text{Math}) = .6$

$P(\text{English}) = .7$

$P(\text{English} | \text{Math}) = .8$

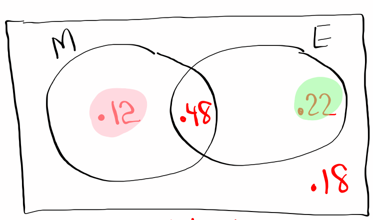
$P(\text{math and English})$

$$P(\text{English}|\text{Math}) = \frac{P(M \text{ and } E)}{P(\text{Math})}$$

$$.8 = \frac{P(M \text{ and } E)}{.6}$$

Cross-Multiply

$\boxed{P(\text{Math and English}) = .48}$



Total = 1

$$P(\text{Math}|\text{English}) = \frac{P(M \text{ and } E)}{P(\text{English})} = \frac{.48}{.7} = \boxed{.686}$$

$P(\text{Math only OR English only}) = .12 + .22 = \boxed{.34}$

SG 13 ✓

Data
- 1) Qualitative
- 2) Quantitative
  - 1) Discrete
    Countable
  - 2) Continuous
    Measureable

Let x be a discrete random variable with Prob. dist. $P(x)$.

What is Prob. distribution? It is a way to give Prob. of all possible outcomes.

1) Form of a table
2) Form of a formula
3) Form of a graph.

1) $0 \le P(x) \le 1$
2) $\sum P(x) = 1$
3) $P(x) = 1 \Leftrightarrow$ Sure event
4) $P(x) = 0 \Leftrightarrow$ Impossible event
5) $0 < P(x) \le .05 \Leftrightarrow$ Rare event

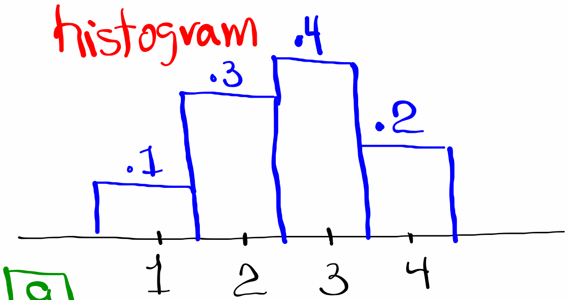
Consider the chart below

x	$P(x)$
1	.1
2	.3
3	.4
4	.2

1) Verify $\sum P(x) = 1$

$.1 + .3 + .4 + .2 = 1 \checkmark$

2) Draw Prob. dist. histogram



3) $P(x \geq 2)$

$= 1 - P(x=1) = 1 - .1 = \boxed{.9}$

4) $P(x \leq 3) = 1 - P(x=4) = 1 - .2 = \boxed{.8}$

Consider the chart below for x & $P(x)$

x	$P(x)$
1	.15
2	.25
3	.35
4	.25

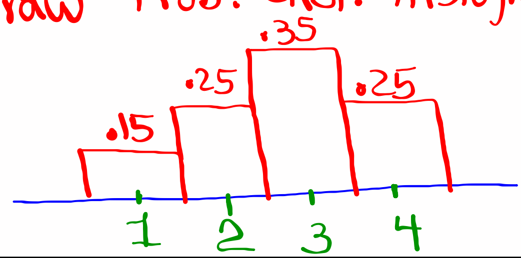
1) Find $P(x=4)$

$= 1 - [.15 + .25 + .35] = \boxed{.25}$

2) $P(x=1 \text{ or } x=4) =$

$.15 + .25 = \boxed{.4}$

3) Draw Prob. dist. Histogram



Complete the chart below

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.2	.2	.2
3	.5	1.5	4.5
5	.3	1.5	7.5

$\sum P(x) = 1$ ✓

$\sum x p(x) = 3.2$

$\sum x^2 p(x) = 12.2$

Mean μ mu $\quad \mu = \sum x p(x) = \boxed{3.2}$

Variance σ^2 sigma^2 $\quad \sigma^2 = \sum x^2 p(x) - \mu^2$
$= 12.2 - 3.2^2 = \boxed{1.96}$

Standard deviation σ sigma $\quad \sigma = \sqrt{\sigma^2} = \sqrt{1.96} = \boxed{1.4}$

Using TI:

$x \to L1$

$P(x) \to L2$

	x	$P(x)$	
L1	1	.2	L2
	3	.5	
	5	.3	

$\boxed{\text{STAT}}\ \boxed{\to}$ CALC $\boxed{\text{1:1-Var Stats}}$

List: L1 $\qquad$ L1, L2

FreqList: L2 $\qquad$ $\boxed{\text{Enter}}$

$\boxed{\text{Calculate}}$

$\mu = \bar{x} = 3.2$

$\sigma = \sigma_x = 1.4$

How to find σ^2

$\boxed{\text{VARS}}\ \boxed{\text{Statistics}}\ \boxed{4: \sigma x}$ $\quad n = 1 \leftarrow$ Total Prob.

$\boxed{x^2}\ \boxed{\text{Enter}}$ 1.96

Convert to reduced fraction

$\boxed{\text{Math}}\ \boxed{1: \blacktriangleright \text{Frac}}\ \boxed{\text{Enter}}$ $\quad \frac{49}{25}$

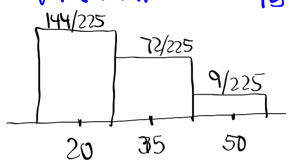
Suppose there are 3 Quarters & 12 dimes.
we randomly pick 2 coins with replacement.

DD → 20¢ → $P(\text{Total}=20¢) = \frac{12}{15}\cdot\frac{12}{15} = \frac{144}{225}$

DQ, QD → 35¢ → $P(\text{Total}=35¢) = \frac{72}{225}$

QQ → 50¢ → $P(\text{Total}=50¢) = \frac{3}{15}\cdot\frac{3}{15} = \frac{9}{225}$

Total	P(Total
20	144/225
35	72/225
50	9/225



Total → L1
P(Total) → L2

Use 1-Var Stats with L1 & L2
$\mu = \bar{x} = 26$
$\sigma = \sigma_x = 8.485$
$n = 1$ Total Prob. = 1

Find σ^2 in reduced fraction

$\sigma^2 = 72$

Expected Value:
25 students, each paid me \$10 to buy a ticket. One ticket is randomly drawn. Winner gets a Calc. worth \$100.
Expected Value per ticket Sold.

Net gain	P(Net gain)
10 − 100	1/25
10 − 0	24/25

Net gain → L1
P(Net gain) → L2
Expected Value/TKT = $\mu = \bar{x}$
1-Var Stats with L1 & L2
E.V. = $\mu = \bar{x} =$ \$6

\$6/TKT

You buy a policy for \$100 to insure your luggage. Any damages, Airline pays you \$1000. Prob. of damage is .2%.

Find expected value per policy sold.

Net gain	P(Net gain)	
100 − 1000	.2% = .002	Damage
100 − 0	99.8% = .998	$\overline{\text{Damage}}$

Net gain → L1
P(Net gain) → L2
1-Var Stats with L1 & L2

SG 14 & SG 15

$E.V. = \mu = \bar{x}$
\$98

Airline makes \$98 per sold policy.

Class QZ 9

x	$P(x)$
1	.05
2	.15
3	.25
4	.35
5	.20

Find

1) $\mu = 3.5 = \boxed{4}$

2) $\sigma = 1.118 = \boxed{1}$

Round to a whole #

3) σ^2 in reduced fraction.
$= 1.25 = \boxed{\frac{5}{4}}$